Unit 3: Linear Functions STUDY GUIDE

1. The function \( a(n) = 3k + 2 \) represents the value of the \( n \)th term in a sequence. What is the sum of the 1st and 4th terms of the sequence?

\[ \begin{align*}
1_{\text{st}} & = 3k + 2 \\
4_{\text{th}} & = 3(4-1)+5 \\
\end{align*} \]

\[ 1_{\text{st}} + 4_{\text{th}} = 5 + 14 = 19 \]

2. What linear equation can be written from the table of values below?

\[ \begin{align*}
M = 1 & \quad (3,3) \\
M = 1 & \quad (4,2) \\
M = 1 & \quad (5,4) \\
M = 1 & \quad (6,5) \\
Y = -2 & \quad (x-3) \\
Y + 3 & = x - 3 \\
Y + 3 & = x - 3 \\
\end{align*} \]

\[ Y = x - 1 \]

3. Water is being pumped into a 10-foot tall cylindrical tank at a constant rate.
- The depth of the water is increasing linearly.
- At 1:30 pm, the water depth was 2.4 feet.
- It is now 4:00 pm, and the depth of the water is 3.9 feet.

What will the depth (in feet) of the water be at 5:00 pm?

\[ 3.9 + 0.5 = 4.5 \text{ ft} \]

4. Ivan compared the slope of the functions \( f(x) \) and \( g(x) \)

\[ \begin{array}{ccc}
\text{ } & f(x) & g(x) \\
\text{Function with } & \text{ } & \text{ } \\
x-intercept at 3 and & \text{ } & \text{ } \\
y-intercept at -2.5. & (3,0) & (0,-3.5) \\
\end{array} \]

What is the slope of the function with the larger slope? Write your answer as a fraction in simplest form.

\[ f(x) \]

\[ m = \frac{3-0}{0-3} = \frac{1}{\text{not possible}} \]

\[ g(x) \]

\[ m = \frac{3-0}{0-3} = \frac{1}{-3} \]

5. Jose began the school year with $400 in his school lunch account.
- The amount of money in the account has decreased linearly.
- After 2 months, he had $330 in his account.
- After 6 months, she had $190 in his account.

Write a function that models the amount of money that Jose has in his account at the end of \( n \) months.

\[ (3,330) \quad (6,190) \]

\[ m = \frac{190-330}{6-3} = \frac{-140}{3} = -\frac{140}{3} \]

\[ y = -35x + 400 \]

6. What is the range of the relation shown below?

\[ \{(1,-2),(-2,0),(-1,2),(1,3)\} \]

\[ \{-2,0,2,3\} \]

7. What is the equation of a line that has a slope of \( -\frac{1}{4} \) and passes through the point \((-3,-2)\)?

\[ \begin{align*}
y + 2 & = -\frac{1}{4}(x - 3) \\
y + 2 & = -\frac{1}{4}x + \frac{3}{4} \\
\end{align*} \]

8. Write a linear equation that passes through the points \((0,-8)\) and \((1,10)\)

\[ \begin{align*}
m & = \frac{10-(-8)}{1-0} = \frac{18}{1} = 18 \\
m & = \frac{1}{1} \\
\end{align*} \]

\[ y = 18x - 8 \]

9. Jack's new car lease costs him $248 a month for a total of 36 months. He put $1,500 down at signing and keeps track each month of the total amount he has spent. Write a function that could represent the total amount of money Tracy has spent on her lease based on the number of months, \( x \), she has paid?

\[ y = 248x + 1500 \]
10. Which equation is represented by the graph?

\[ m = -2 \]
\[ b = 3 \]
\[ y = -3x + 3 \]

- \( a) \) \( 2x - y = 3 \)
- \( b) \) \( 2x + y = 3 \)
- \( c) \) \( 2x + y = -3 \)
- \( d) \) \( 2x - y = -3 \)

11. Two stores have movies to rent.
- The first store charges a $12 per month membership fee plus $2.50 per movie rented. \( y = 2.5x + 12 \)
- The second store has no membership fee but charges $4.50 per movie rented. \( y = 4.50x \)

What is the minimum number of movies a person would need to rent in a month for the first store to be a better deal? \( \text{FIRST STORE \ L < \text{SECOND STORE}} \)

7 MOVIES

12. Which table of values represents a linear function?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3.375</td>
</tr>
<tr>
<td>0</td>
<td>4.5</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

13. Which of the following describes the domain and range of the function in the graph?

- \( a) \) \( D: \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \} \)
  \( R: \{1, 2, 3, \ldots \} \)
- \( b) \) \( D: -4 \leq x \leq 4 \)
  \( R: 1 \leq x \leq 3 \)
- \( c) \) \( D: [-4, -2, 0, 2, 4] \)
  \( R: \{1, 3\} \)
- \( d) \) \( D: -\infty \leq x \leq \infty \)
  \( R: -\infty \leq y \leq \infty \)

14. The total cost, in dollars, of membership in a basketball league is given by the function \( (m) = 35m + 60 \), where \( m \) is the number of months a person is a member. In dollars, how much is the cost of a membership for 1 year?

\[ \frac{35(12) + 60}{12} = 480 \]

15. Oscar planted a tomato seed in his garden. Each day he recorded the height of the tomato plant.

During which interval did the tomato plant grow the:
- Slowest? \( \text{DAYS 3-4, 4-10, 10-12, and 12-14} \)
- Fastest? \( \text{DAY 8 TO DAY 10} \)
16. The function \( f(x) = -0.5x + 10 \) models the height of a candle \( x \) seconds after it is lit. What is the meaning of the \( y \)-intercept of the function?

a) the rate at which the candle is burning  
b) the final height of the candle  
c) the amount of time it will take the candle to burn  
d) the initial height of the candle

17. Which function has the largest value for \( f(-4) \)?

\[
\begin{align*}
f(x) &= 3x - 4 \quad \rightarrow \quad -11 \hline
f(x) &= 5 - 3x \quad \rightarrow \quad 17 \\
f(x) &= 10 - 2^x \quad \rightarrow \quad 9.9815 \\
f(x) &= 3^x + 15 \quad \rightarrow \quad 15.01
\end{align*}
\]

18. The table below shows the hours, \( x \), spent working on a new road and the distance, \( y \), of the finished road.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>1.5</td>
</tr>
<tr>
<td>200</td>
<td>6</td>
</tr>
<tr>
<td>250</td>
<td>10.5</td>
</tr>
<tr>
<td>300</td>
<td>12</td>
</tr>
<tr>
<td>350</td>
<td>19.5</td>
</tr>
</tbody>
</table>

What is the slope of the data? \( m = 0.3 \)

19. Jonathan opens a savings account and deposits some money in the account every month. The table below shows the value of his savings account over time.

<table>
<thead>
<tr>
<th>Time (in months)</th>
<th>Amount (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>160</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>260</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
</tr>
<tr>
<td>7</td>
<td>380</td>
</tr>
<tr>
<td>8</td>
<td>460</td>
</tr>
<tr>
<td>9</td>
<td>560</td>
</tr>
<tr>
<td>10</td>
<td>680</td>
</tr>
</tbody>
</table>

Over which interval of time can the relation in the table be modeled by a linear function? \( \text{Month 1 to Month 4} \)

20. If \( g(x) = -3x + 1 \), what is the value of \( g(-2) \)?

\[
g(-2) = -3(-2) + 1 = 7
\]

21. A sequence is defined recursively as follows.

\[
a_1 = 2.5 \\
a_n = -1 + a_{n-1}
\]

What are the first five terms of this sequence? \( 2.5, 1.5, 0.5, -0.5, -1.5 \)

22. Examine the three graphs shown.

Which graphs have the same domain? \( \text{Graphs 1 and 2} \)

Which graphs have the same range? \( \text{Graphs 2 and 3} \)
23. Jason studied how quickly ants consume things. He counted the number of ants on a banana peel at the end of each minute for five minutes. His results formed the pattern below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_n$</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>33</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

Let $n$ represent the number of minutes since Jason began his study. Which expression could be used to predict the number of ants on the banana peel after $n$ minutes?

- $A_n = 7(n-1) + 12$
- $A_n = 7n - 7 + 12$
- $A_n = 7n + 5$

24. Which problem could be modeled by the graph shown below?

![Graph showing savings over weeks]

a. John has $3 saved and saves $5 more each week. How much money will he have saved after $x$ weeks?

b. John has $3 saved and spends $5 a week. How much money will he have left after $x$ weeks?

C. John has $5 saved and saves $3 more each week. How much money will he have saved after $x$ weeks?

d. John has $5 saved and spends $3 each week. How much money will he have left after $x$ weeks?

25. The table below shows the cost to rent a movie for different numbers of days at a movie rental store.

<table>
<thead>
<tr>
<th>Days</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2</td>
<td>$6.00</td>
</tr>
<tr>
<td>3</td>
<td>$6.00</td>
</tr>
<tr>
<td>+1</td>
<td>$8.50</td>
</tr>
<tr>
<td>5</td>
<td>$8.50</td>
</tr>
<tr>
<td>+3</td>
<td>$9.75</td>
</tr>
<tr>
<td>6</td>
<td>$9.75</td>
</tr>
<tr>
<td>+1</td>
<td>$13.50</td>
</tr>
<tr>
<td>9</td>
<td>$13.50</td>
</tr>
</tbody>
</table>

What is the meaning of the rate of change for the data?

- The cost to rent a movie increases by $1.25 for each additional day the movie is rented.
- The cost to rent a movie increases by $2.00 for each additional day the movie is rented.
- The cost to rent a movie increases by $2.50 for each additional day the movie is rented.
- The cost to rent a movie increases by $2.75 for each additional day the movie is rented.